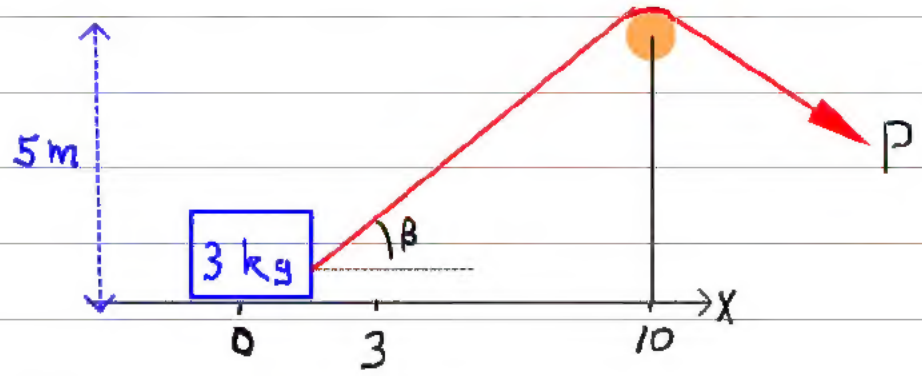
Meriam 3/6 : Work & Energy

$$T_2 = T_1 + U_{1\to 2}$$

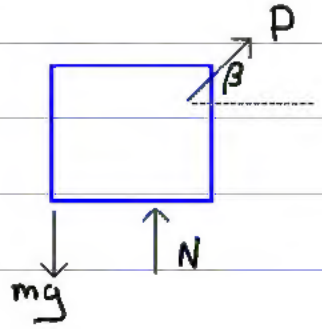
- Relates speeds of a mass at the begining and end of a trip, based on work done

Eg.
friction = 0
P = 10 Newtons
$\vec{V}_1 = -4\hat{i}$ m/s



Find v when box reaches x = 3m

- only tension in rope does work
- angle β of rope keeps changing



at x = 3 at x = 0

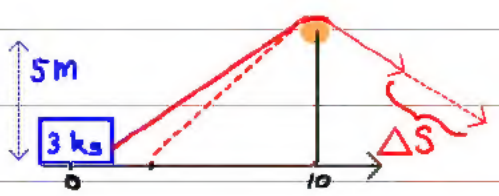
$$T_2 = T_1 + U_{1\to 2}$$

$$(\tfrac{1}{2})(3)(V_2)^2 = (\tfrac{1}{2})3)(-4)^2 + (P)(\Delta S)$$

10

where

$$\Delta S = \sqrt{10^2+5^2} - \sqrt{7^2+5^2} = 2.58$$



→ $V_2 = +/- 5.76$ → $V_2 = 5.76$ m/s

(no direction info from energy eqn)

Eg.

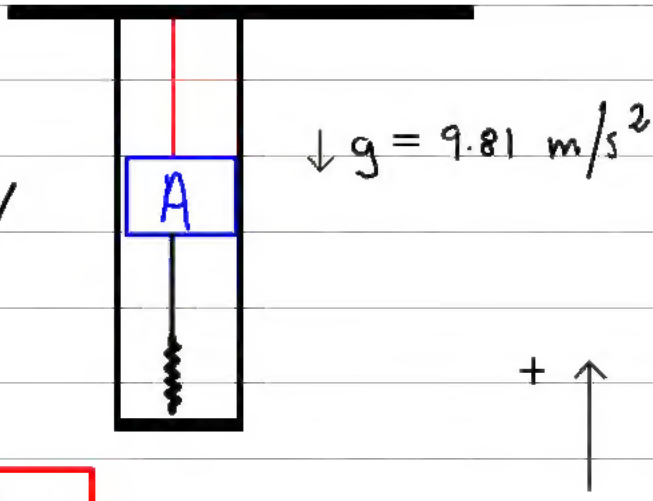
Given:

$V_0 = 0$

Initial tension in rope $= 15\ N$

$k = 1400\ N/m$

$A = 3\ kg$



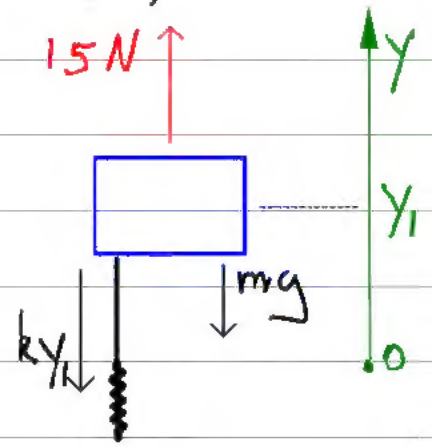
(a) Find initial deformation of spring

- Draw FBD
- Define coordinate system
- Specify origin

★ $y = 0$ is position at which spring is relaxed

→ $F_{spring} = -ky$

★ Define y_1 = initial position of A (may be +/−)



$\Sigma F_y = 0$

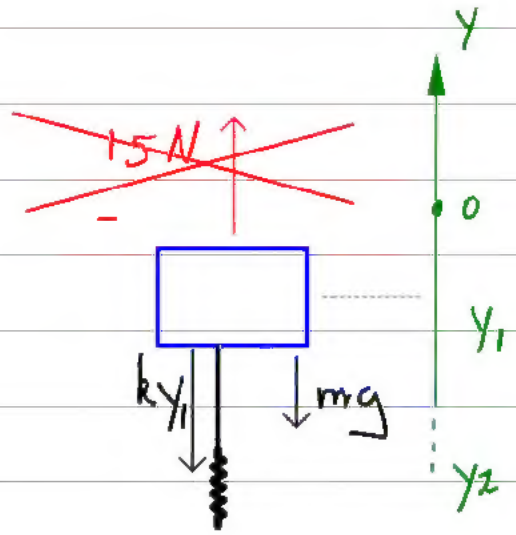
$-(3kg)(9.81) - 1400\,y_1 + 15 = 0$

→ $y_1 = -0.0103$ meters

(initial spring force $= -(1400)(-0.0103) = +14.4\,\hat{\jmath}\,N$

(b) How far does A drop if string is cut?

→ Mass A goes on a trip from position 1 at y_1, to position 2 at y_2



Note: $\vec{a}$ will vary with position

$$T_2 = T_1 + U_{1\to 2}$$

$$0 = 0 - mg\,\Delta h - \tfrac{1}{2}k\left(y_2^2 - y_1^2\right)$$

where $\Delta h = y_2 - y_1$

$$0 = 0 - (3)(9.81)\left(y_2 - (-0.0103)\right)$$

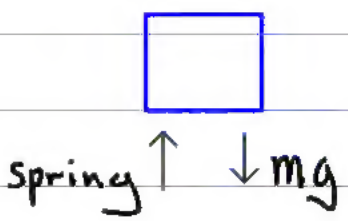
$$-\left(\tfrac{1}{2}\right)(1400)\left(y_2^2 - (-0.0103)^2\right)$$

→ $y_2 = -0.0103\ \text{m}$ or **$y_2 = -0.031\ \text{m}$**

$$y_2 - y_1 = -0.031 - (-0.0103)$$

$$\Delta y = -0.021\ \text{m}$$

How would you find V_{max}?



Eg:

Given:

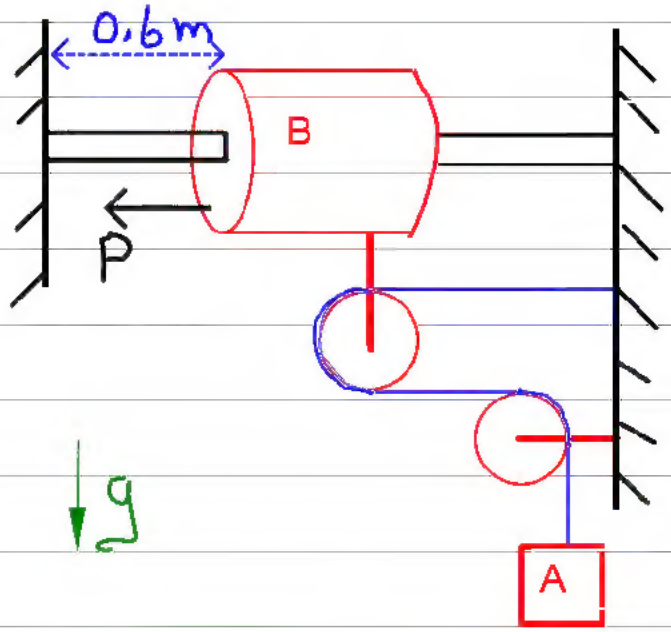
P = 150 Newtons

A = 3 kg

B = 8 kg

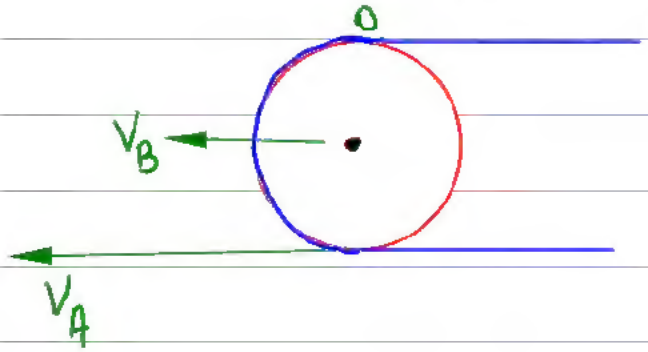
$V_{initial}$ 0

zero friction



Find v at which B hits left wall.

Note upper wheel:



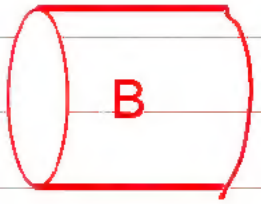
$$V_B = \frac{V_A + 0}{2}$$

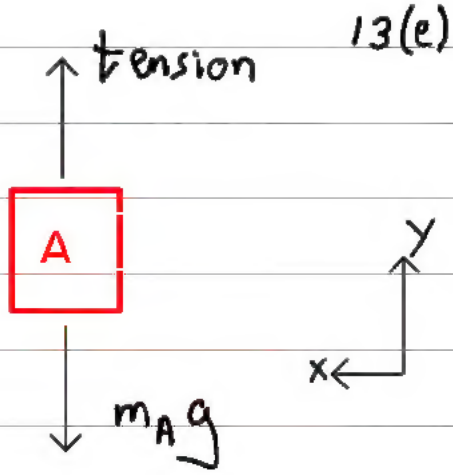
$$V_A = 2 V_B$$

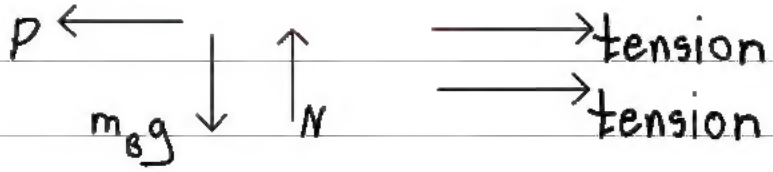
→ A rises 1.2 m as B moves 0.6 m

13(e)

FBD

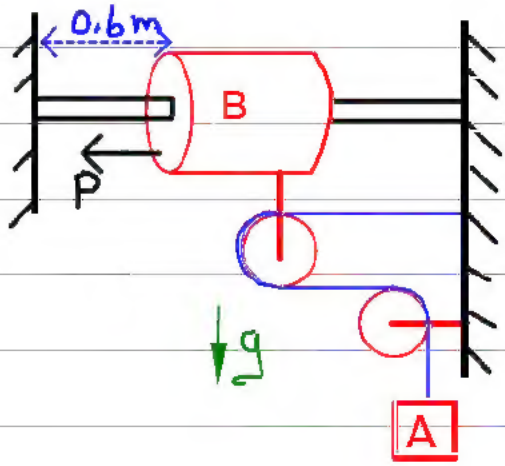






① = initial stationary position

② = instant that B hits the wall



$$T_2 = T_1 + U_{1\to 2}$$

$$\tfrac{1}{2} m_A (V_A)_2^2 + \tfrac{1}{2} m_B (V_B)_2^2 =$$

($V_A = 2V_B$)

$$0 + (P)(0.6) - (m_A)(g)(\Delta h) - (2)(\text{tension})(0.6) + (\text{tension})(1.2)$$

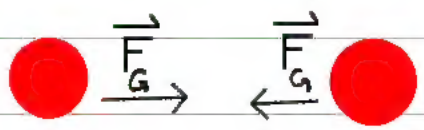
$P = 150$, $m_A = 3$, $\Delta h = 1.2$

$$\rightarrow \quad (V_B)_2^2 = 5.47$$

$$\boxed{(V_B)_2 = 2.34 \text{ m/s}}$$

Does total internal work always equal 0?

No



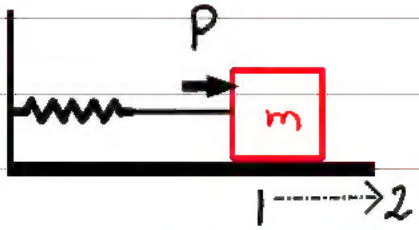
Summary

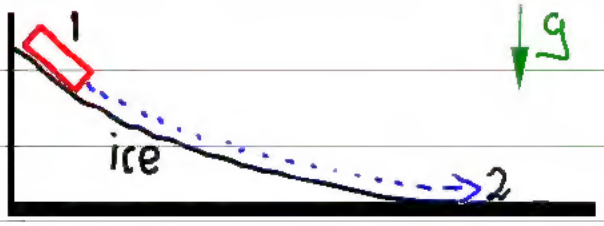
- For "snapshot" kinetics problems, use $\vec{F} = m\vec{a}$
- But if particle moves over a period of time from position ① ⟶ position ②, obtain V_2 using :

$$\vec{V}_2 = \vec{V}_1 + \int_1^2 \vec{a}\, dt = \vec{V}_1 + \int_1^2 \vec{F}/m\, dt$$

or $T_2 = T_1 + U_{1\to 2}$, where $T = \frac{1}{2} m v^2$

e.g.





For next Lecture

- Read 3/7 : Potential Energy